

much O^{18} as the original phosphate. This value agrees well with the 25 per cent value predicted by the proposed enzymatic sequence for thiosulfate oxidation.

The phosphate, arsenate, and O^{18} data do not exclude PAPS as an intermediate in the oxidation of thiosulfate. However, the unique occurrence of APS reductase in high specific activity as well as the low specific activity of PAPS reductase suggest that APS is the product of sulfite oxidation and PAPS is probably not an intermediate in the oxidation of thiosulfate.

Summary.—Cell-free extracts of *Thiobacillus thioparus* that will oxidize thiosulfate have been prepared. The metabolism of sulfur-containing nucleotides in these extracts has been investigated and high levels of the enzymes adenosine triphosphate sulfurylase, adenosine diphosphate sulfurylase, and adenosine 5'-phosphosulfate reductase observed. From these observations a pathway of thiosulfate oxidation is proposed that involves adenosine 5'-phosphosulfate as an intermediate.

I should like to express my appreciation to Dr. G. David Novelli for his advice and encouragement during the course of this investigation.

* Operated by Union Carbide Corp. for the U.S. Atomic Energy Commission.

¹ Vishniac, W., *J. Bacteriol.*, **64**, 363 (1952).

² Vishniac, W., and M. Santer, *Bacteriol. Rev.*, **21**, 195 (1957).

³ Santer, M., M. Margulies, N. Klinman, and R. Kaback, *J. Bacteriol.*, **79**, 313 (1960).

⁴ Santer, M., *Biochem. Biophys. Research Comm.*, **1**, 9 (1959).

⁵ Lipmann, F., *Science*, **128**, 575 (1958).

⁶ Wilson, L. G., and R. S. Bandurski, *J. Am. Chem. Soc.*, **80**, 5576 (1958).

⁷ Hilz, H., M. Kittler, and G. Knape, *Biochem. Z.*, **332**, 151 (1959).

⁸ Peck, H. D., Jr., these PROCEEDINGS, **45**, 701 (1959).

⁹ Peck, H. D., Jr. (to be published).

¹⁰ Starkey, R. L., *J. Bacteriol.*, **28**, 365 (1934).

¹¹ Baddiley, J., J. G. Buchanan, and R. Letters, *J. Chem. Soc.*, 1076 (1957).

¹² Robbins, P. W., and F. Lipmann, *J. Biol. Chem.*, **233**, 681 (1958).

¹³ Wilson, L. G., and R. S. Bandurski, *J. Biol. Chem.*, **233**, 975 (1958).

¹⁴ Kaji, A., and W. D. McElroy, *J. Bacteriol.*, **77**, 630 (1959).

¹⁵ Slein, M. W., G. T. Cori, and C. F. Cori, *J. Biol. Chem.*, **186**, 763 (1950).

¹⁶ Parker, C. D., and J. Prisk, *J. Gen. Microbiol.*, **8**, 344 (1953).

¹⁷ Suzuki, I., and C. H. Werkman, *Biochem. J.*, **74**, 359 (1960).

¹⁸ Lange-Posdecva, I. P., *Arch. sci. biol. (U.S.S.R.)*, **30**, 189 (1930).

¹⁹ Heinberg, M., I. Fridovich, and P. Handler, *J. Biol. Chem.*, **204**, 913 (1953).

²⁰ Cohn, M., *J. Cellular Comp. Physiol.*, **54** (Supplement 1), 17 (1959).

ON THE INTERACTION OF COLLOIDAL PARTICLES

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Introduction.—The theory of the stability of lyophobic colloidal systems has focused attention on the interaction energy between colloidal particles. To account for such phenomena as flocculation, for example, some long-range interaction is required, and following a suggestion of Kallmann and Willstätter,¹ Hama-

ker² and de Boer³ investigated the character of the van der Waals (London) dispersion energy between colloidal particles: the same problem had earlier been discussed by Bradley.⁴ Hamaker treated spherical colloidal particles assuming the additivity of dispersion energies and showed that although the range of the atomic dispersion energies was of the order of atomic dimensions, the additivity of the dispersion energies led for colloidal particles to an interaction range of the order of colloidal dimensions. The original interest in spherical particles has continued⁵ and other figures have also been treated.

The problem is easily stated: one assumes that the atoms are distributed according to a function, ρ , of position; the interaction of pairs of atoms is taken to be represented by a pair potential $\phi(r_{ij})$, where r_{ij} is the distance separating atom i from atom j and $\phi(r_{ij})$ is taken most conveniently as some inverse power potential, r_{ij}^{-n} ; the double-sum over both particles (additivity) which could be written down amenable to machine computation, is here replaced by an integral over the volumes of both particles assuming the function ρ to be continuous; thus the total interaction potential between a pair of colloidal particles can be written as

$$\Phi = \int_{v_1} \int_{v_2} \rho_1 \rho_2 \phi(r) dv_1 dv_2. \quad (1)$$

In addition to spheres^{2, 4, 5} the integral of equation (1) has been done exactly for the case of circles,^{6, 7} both in coplanar and sandwich-like configurations,⁸ and for spherical shells.⁹

Other figures have been treated by approximate methods and one can mention the case of certain rectangular parallelepipeds and ellipsoids treated by Vold¹⁰ and of infinite plates by de Boer.³ Most recently Sparnaay¹¹ has used the methods of Bouwkamp (circular disks) to treat parallel and crossed cylinders; the elegant Bouwkamp procedure is exact but in the former case Sparnaay treats one cylinder as infinite and in the second case both cylinders as infinite.

In all cases the problem reduces to the integral described by equation (1) for which the London-van der Waals potential is written as

$$\phi(r) = (-\lambda)/r^6 \quad (2)$$

where λ is a constant given in the first approximation by $\lambda = 3\alpha^2 h\nu_0/4$ (α = polarizability of the atom; $h\nu_0$, a characteristic energy corresponding to the chief specific frequency ν_0 taken from the dispersion spectrum of the atom); other forms have been proposed for λ by Slater and Kirkwood¹² and by Neugebauer¹³ which generally lead to higher values than those computed from the quoted result. The choice of r^{-6} needs examination in the case that the particles are separated by a distance comparable to the magnitude of the London frequency, for then, as shown by Casimir and Polder,¹⁴ the finite time of propagation leads to an attenuation of the energy from r^{-6} to r^{-7} at distances of the order of 10^3 Å, viz., distances of the order of colloidal size.

Hamaker² has also shown that even in the case where the colloidal particles are immersed in a fluid with which they interact strongly, the net force between the colloidal particles remains attractive for identical particles. This theorem is proved under the assumption that the interaction energy is decomposable into an energy independent of the orientation of the fluid molecules and a usually neglected term depending upon fluid orientation. The modification of the de Boer-Hamaker

theorem necessary communication.¹⁵ interaction has re workers.¹⁶

We will treat in similar to some pre lar figures are gen "crossed" configur range approximat tend towards r^{-6}

Calculations.—C rated by a distanc the attractive ene proportional to r^{-1}

Hence $\lambda = 2\epsilon r_0^6$ w r^0 . Next we define

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The integrations point x_1 in one rod, finally integrating in this case one-dim

$$K_6 = \int_0^a dx_1$$

$$K_{12} = \int_0^a dx_1 \int_0^d dx_2$$

It is obvious tha limits of integratio (3) for large d , but

theorem necessary when the latter term is included will be discussed in a separate communication.¹⁵ The question of the specificity of the London-van der Waals interaction has received a good deal of attention lately, mostly by Jehle and co-workers.¹⁶

We will treat in this communication the integral of equation (1) for several cases similar to some previously described with approximate calculations. Our rectangular figures are general and our method exact; however, we shall not treat the case of "crossed" configurations. We shall discuss and compare our results for the short-range approximation; it is clear that in the long-range approximation all our results tend towards r^{-6} as is easily shown by appropriate Taylor expansions.



FIG. 1

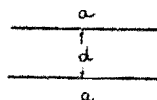


FIG. 2

Calculations.—Case 1. Rods. Imagine two colinear thin rods of length a separated by a distance d (Fig. 1). In this case and the one that follows we shall treat the attractive energy according to equation (2) and the repulsion by a potential proportional to r^{-12} , both energies chosen to satisfy the Lennard-Jones potential

$$\phi(r) = \epsilon \left\{ \left(\frac{r_0}{r} \right)^{12} - 2 \left(\frac{r_0}{r} \right)^6 \right\} \quad (3)$$

Hence $\lambda = 2\epsilon r_0^6$ where $-\epsilon$ is the value of ϕ at the equilibrium interatomic distance r_0 . Next we define

$$K_6 \equiv \iint \frac{1}{r^6} dV_1 dV_2$$

$$K_{12} \equiv \iint \frac{1}{r^{12}} dV_1 dV_2 \quad (4)$$

in order that

$$\Phi^* = r_0^{12} K_{12} - 2r_0^6 K_6 \quad (5)$$

where for convenience we set $\rho^2 = 1$ and use the reduced potential $\Phi^* = \Phi/\epsilon$.

The integrations indicated for K_6 and K_{12} are easily completed upon picking a point x_1 in one rod, integrating the interaction of this point with the other rod and finally integrating x_1 over the length of the first rod (the differential volumes are in this case one-dimensional). We obtain

$$K_6 = \int_0^a dx_1 \int_{d+a-x_1}^{d+2a-x_1} x^{-6} dx = \frac{1}{20} \left(\frac{1}{d^4} + \frac{1}{(d+2a)^4} - \frac{2}{(d+a)^4} \right) \quad (6)$$

$$K_{12} = \int_0^a dx_1 \int_{d+a-x_1}^{d+2a-x_1} x^{-12} dx = \frac{1}{110} \left(\frac{1}{d^{10}} + \frac{1}{(d+2a)^{10}} - \frac{2}{(d+a)^{10}} \right) \quad (7)$$

It is obvious that rods of unequal lengths may be easily treated by changing the limits of integration in equations (6) and (7). The potential reduces to equation (3) for large d , but for $d \ll a$ we find approximately

$$\Phi^* \simeq \frac{r_0^{12}}{110d^{10}} - \frac{r_0^6}{10d^4} \quad (8)$$

showing that as long as $d \ll a$ we have an interaction which is two orders lower than the interatomic potential and which is in addition independent of the lengths of the rods.

It is also possible to consider the equilibrium separation of the rods by solving $(\partial\Phi^*/\partial d) = 0$ for d_0 to obtain $d_0 \simeq 0.781 r_0$: in other words, the equilibrium inter-rod distance is a little over three-quarters of the equilibrium interatomic distance, and this effect is certainly due to the additivity of the energy.

We next treat thin rods which are aligned and separated by a distance d (Fig. 2). As before we choose a point in one rod, find its energy of interaction with the second rod and then integrate the chosen point over the first rod to obtain the total potential. Again note that in what follows rods of unequal lengths can be treated by appropriate changes in the limits of integration.

K_6 and K_{12} become:

$$K_6 = \int_0^a dx_1 \int_{-x_1}^{a-x_1} \frac{dx}{(x^2 + d^2)^3} = \frac{1}{4d^4} - \frac{1}{4d^2(a^2 + d^2)} + \frac{3a}{4d^5} \tan^{-1} \frac{a}{d}, \quad (9)$$

$$K_{12} = \int_0^a dx_1 \int_{-x_1}^{a-x_1} \frac{dx}{(x^2 + d^2)^6} = \frac{187}{640d^{10}} - \frac{1}{40d^2(d^2 + a^2)^4} - \frac{3}{80d^4(d^2 + a^2)^3} - \frac{21}{320d^6(d^2 + a^2)^2} - \frac{21}{128d^8(d^2 + a^2)} - \frac{63a}{128d^{11}} \tan^{-1} \frac{a}{d}. \quad (10)$$

It is interesting to notice the influence of the repulsive potential in this case as compared to the colinear case; the effect is not unexpected since the repulsion falls off more rapidly than does the attraction with increasing distance.

In analogy with the previous example we compute the potential for $d \ll a$ to get

$$\Phi^* \simeq \frac{r_0^{12}}{128d^{11}} \left(63 a \tan^{-1} \frac{a}{d} \right) - \frac{r_0^6}{2d^5} \left(3 a \tan^{-1} \frac{a}{d} \right) \quad (11)$$

showing that the interaction is down only one order from the interatomic potential and becomes proportional to the length of the rod. If we approximate $\tan^{-1} a/d$ by $\pi/2$, we obtain for d_0 the value $d_0 \simeq 0.947 r_0$; thus the decrease in intermolecular distance from the interatomic distance r_0 is in this case approximately one fourth that found for colinear rods. The quantity r_0 is related to the so-called van der Waals radius of the atoms in question, and one implication of the above results is that aggregates of atoms tend to come to equilibrium distances which are closer than their "free-atom" contact radii. These remarks depend for their complete validity on the constancy of the function ρ . It is obvious that for close approach ρ no longer remains constant and we may take these results, therefore, as qualitative and describing a mathematical problem with no obvious physical counterpart.

We also notice that in the lowest limit $K_6 \simeq d^{-5}$ in exact keeping with Sparnaay's¹¹ result for infinite thin cylinders. The factor of pi occurring in previous results stems from their implicit expansion of the arctangent term which appears in the exact answer.

Case 2. Rectangles: We shall calculate here, and in the cases to follow, only K_6 since, as will soon be seen, the calculation of K_{12} is prohibitively tedious; and in



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$$K_6 = \int_0^b dy_1 \int_{-y_1}^b$$

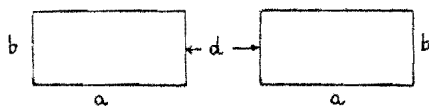


FIG. 3

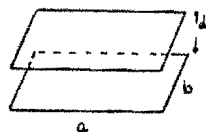


FIG. 4

any case for colloidal systems it is more appropriate to consider the double-layer interaction in the manner of Hamaker when the total potential is required.

Consider two coplanar, parallel rectangles of sides a and b and separated by a distance d (Fig. 3). It is clear from the previous cases that

$$K_6 = \int_0^a dx_1 \int_0^b dy_1 \int_{d+a-x_1}^{d+2a-x_1} dx \int_{-y_1}^{b-y_1} \frac{dy}{(x^2 + y^2)^3} \quad (12)$$

where x_1 and y_1 are the coordinates of a point chosen in either one of the interacting rectangles. The energy of interaction of this point and the second rectangle is proportional to

$$\int_{d+a-x_1}^{d+2a-x_1} dx \int_{-y_1}^{b-y_1} (x^2 + y^2)^{-3} dy$$

and the total potential is found by integrating this quantity over the coordinates of the second rectangle to give equation (12). The integrations are more easily performed in the following equivalent fashion:

$$K_6 = \int_0^a dx_1 \int_{d+a-x_1}^{d+2a-x_1} dx \int_0^b dy_1 \int_{-y_1}^{b-y_1} (x^2 + y^2)^{-3} dy. \quad (12a)$$

We define

$$H(d) \equiv -\frac{1}{48d^2} + \frac{d}{16b^3} \tan^{-1} \frac{d}{b} + \frac{b}{16d^3} \tan^{-1} \frac{b}{d} \quad (13)$$

and write the final answer for K_6 as

$$K_6 = H(d) + H(d+2a) - 2H(d+a). \quad (14)$$

When d is small compared to the shorter dimension of the rectangle, then we get:

$$K_6 \simeq H(d) \simeq \frac{\pi b}{32d^3} \quad (15)$$

showing an inverse cubic dependence on d and a linear dependence on b , the length of the adjacent sides.

Analogous to the case of rods we will next treat the case of parallel aligned rectangles (Fig. 4). Again choosing a point (x, y) and calculating its potential with respect to the second rectangle, we then integrate x and y over the coordinates of the first rectangle to obtain K_6 :

$$K_6 = \int_0^b dy_1 \int_{-y_1}^{b-y_1} dy \int_0^a dx_1 \int_{-x_1}^{a-x_1} (x^2 + y^2 + d^2)^{-3} dx$$

$$K_6 = \int_0^b dy_1 \int_{-y_1}^{b-y_1} dy \left\{ \frac{3a}{4(y^2 + d^2)^{5/2}} \tan^{-1} \frac{a}{(y^2 + d^2)^{1/2}} + \frac{1}{4(y^2 + d^2)^2} - \frac{1}{4(y^2 + d^2)(a^2 + y^2 + d^2)} \right\}^*$$

$$K_6 = \left(\frac{bd^2 + 2a^2b}{2d^4(d^2 + a^2)^{1/2}} \right) \tan^{-1} \frac{b}{(d^2 + a^2)^{1/2}} + \left(\frac{ad^2 + 2ab^2}{2d^4(d^2 + b^2)^{1/2}} \right) \times \\ \tan^{-1} \frac{a}{(d^2 + b^2)^{1/2}} - \frac{b}{2d^3} \tan^{-1} \frac{b}{d} - \frac{a}{2d^3} \tan^{-1} \frac{a}{d}. \quad (16)$$

For the special case of d small relative to both dimensions of the interacting rectangles we have:

$$K_6 \simeq \frac{ab}{d^4} \left(\tan^{-1} \frac{b}{a} + \tan^{-1} \frac{a}{b} \right) = \frac{\pi ab}{2d^4}. \quad (17)$$

It is clear now that the pattern involves a power dependence (at close distances) which is always one higher for the case where the figures are aligned, than that for which they are parallel. This is due to the fact that the aligned configuration tends to emphasize the repulsive part of the potential relative to the attractive part in accordance with distance and additivity.

Case 3. Parallelepipeds: Our treatment of parallelepipeds overlaps the papers of Vold¹⁰ and Sparnaay,¹¹ however, both previous workers did only the first two integrations exactly; after completing the first two, Vold specialized to identical particles and then expanded the integrand in a power series retaining only the first term. The subsequent computations retain validity for particles of mean diameter small compared to their length and separated by distances also small compared to their length. Vold remarks that the method is applicable to higher order terms and could be extended with some effort.

We shall describe the interaction of two aligned right parallelepipeds of dimensions $a \times b \times c$ separated by a distance d (Fig. 5). By our standard recipe we write

$$K_6 = \int_0^c \int_{d+c-z_1}^{d+2c-z_1} \int_0^b \int_{-y_1}^{b-y_1} \int_0^a \int_{-z_1}^{a-z_1} \frac{dx dx_1 dy dy_1 dz dz_1}{(x^2 + y^2 + z^2)^3}. \quad (18)$$

The first four integrations have already been done in connection with the previous problem and result in equation (16). Introducing equation (16) and completing the fifth integration we arrive at

$$K_6 = \int_0^c dz_1 \left[\frac{d+2c-z_1}{d+c-z_1} \frac{b}{4z^2} \tan^{-1} \frac{b}{z} + \frac{1}{4b} \tan^{-1} \frac{b}{z} - \frac{1}{4z} - \right. \\ \left. \frac{b(z^2 + a^2)^{1/2}}{2a^2z} \tan^{-1} \frac{b}{(z^2 + a^2)^{1/2}} - \frac{b^2}{2a^2(a^2 + b^2)^{1/2}} \tan^{-1} \frac{z}{(a^2 + b^2)^{1/2}} + \right. \\ \left. \left(\frac{2b(z^2 + a^2)^{1/2}}{3a^2z} - \frac{b(z^2 + a^2)^{1/2}}{3z^3} \right) \tan^{-1} \frac{b}{(z^2 + a^2)^{1/2}} + \right. \\ \left. \frac{2b^2}{3a^2(a^2 + b^2)^{1/2}} \tan^{-1} \frac{z}{(a^2 + b^2)^{1/2}} + \frac{b^2}{3z(a^2 + b^2)} + \right. \\ \left. \frac{b^2}{3(a^2 + b^2)^{3/2}} \tan^{-1} \frac{z}{(a^2 + b^2)^{1/2}} + \text{terms with } a, b \text{ interchanged} \right]. \quad (19)$$

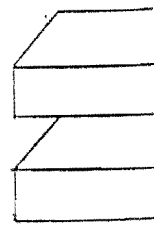


Fig. 5

Collecting all of integration

$$K_6 = \int_{d+c}^{d+2c} \frac{(a^2 + b^2)^{1/2}}{6a} \dots$$

Finally,

$$K_6 = \left[\frac{d+2c}{d+c} \dots \right] \left(\frac{z^2 - b^2}{4bz} \right) \tan^{-1} \dots$$

It is of interest to note that the expression becomes positive

which is described by Vold. It is also recovered by Vold.

We complete the cross-section by restricting ourselves to the case of aligned rectangles.

Again we restrict ourselves to the case of aligned rectangles and consider the case of aligned rectangles.

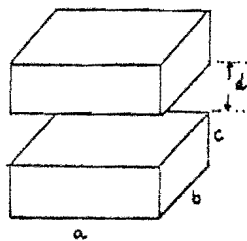


FIG. 5

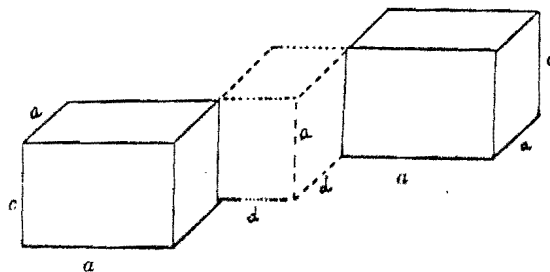


FIG. 6

Collecting all terms and noting that all the functions are odd, we rewrite the limits of integration on the final integral to obtain

$$K_6 = \int_{d+c, d+c}^{d+2c, d} dz \left(\frac{a}{4z^2} + \frac{1}{4a} \right) \tan^{-1} \frac{a}{z} + \left(\frac{b}{4z^2} + \frac{1}{4b} \right) \tan^{-1} \frac{b}{z} - \frac{1}{6z} + \frac{(a^2 + b^2)^{3/2}}{6a^2b^2} \tan^{-1} \frac{z}{(a^2 + b^2)^{1/2}} + (z^2 + a^2)^{1/2} \left(\frac{b}{6za^2} - \frac{b}{3z^3} \right) \times \tan^{-1} \frac{b}{(z^2 + a^2)^{1/2}} + (z^2 + b^2)^{1/2} \left(\frac{a}{6zb^2} - \frac{a}{3z^3} \right) \tan^{-1} \frac{a}{(z^2 + b^2)^{1/2}}. \quad (20)$$

Finally,

$$K_6 = \left[\int_{d+c, d+c}^{d+2c, d} \frac{1}{4} \ln \left(\frac{z^4 + z^2a^2 + z^2b^2 + a^2b^2}{z^4 + z^2a^2 + z^2b^2} \right) + \left(\frac{z^2 - a^2}{4az} \right) \tan^{-1} \frac{a}{z} + \left(\frac{z^2 - b^2}{4bz} \right) \tan^{-1} \frac{b}{z} + \frac{z(a^2 + b^2)^{3/2}}{6a^2b^2} \tan^{-1} \frac{z}{(a^2 + b^2)^{1/2}} + \left(\frac{1}{6z^2} + \frac{1}{6a^2} \right) b(z^2 + a^2)^{1/2} \tan^{-1} \frac{b}{(z^2 + a^2)^{1/2}} + \left(\frac{1}{6z^2} + \frac{1}{6b^2} \right) a(z^2 + b^2)^{1/2} \tan^{-1} \frac{a}{(z^2 + b^2)^{1/2}} \right]. \quad (21)$$

It is of interest to consider the case of equation (21) when d is small. It then becomes possible to compress equation (21) to the result

$$K_6 \simeq \frac{\pi ab}{12d^2} \quad (22)$$

which is identical to the well-known result for parallel plates and to the case described by Vold as equation (5a). By suitable manipulation, Vold's equation (5) is also recoverable wherein the approximations become clear.

We complete these calculations by describing skew parallelepipeds of square cross-section (Fig. 6). The details of this calculation are tedious and we shall restrict ourselves to presenting only a sketch of procedure.

Again we select a random point (in one parallelepiped) of coordinates (x_1, y_1, z_1) and consider its interaction with the second parallelepiped; this result is then

integrated over the first parallelepiped. Using a more convenient order of integration we have

$$K_6 = \int_0^a dx_1 \int_{d+a-x_1}^{d+2a-x_1} dx \int_0^a dy_1 \int_{d+a-y_1}^{d+2a-y_1} dz \int_0^c dz_1 \int_{-z_1}^{c-z_1} dz (x^2 + y^2 + z^2)^{-2}. \quad (23)$$

From the previous problem we know the first four integrations; thus,

$$K_6 = \iint \left[\frac{d+2a,d}{d+a,d+a} - \frac{y}{4x^3} \tan^{-1} \frac{y}{x} + \left(\frac{2cy^2 + x^2y}{4x^4(c^2 + x^2)^{1/2}} \right) \times \right. \\ \left. \tan^{-1} \frac{y}{(c^2 + x^2)^{1/2}} + \frac{2cy^2 + cx^2}{4x^4(x^2 + y^2)^{1/2}} \tan^{-1} \frac{c}{(x^2 + y^2)^{1/2}} \right] dx dy. \quad (24)$$

By analogy with equation (21) we have for the term in d (there are also terms in $d+a$ and $d+2a$):

$$K_6(d) = \left[\frac{d+2a,d}{d+a,d+a} \frac{1}{8} \ln \frac{d^2 + x^2}{c^2 + d^2 + x^2} + \frac{1}{8} \left(\frac{x}{d} - \frac{d}{x} \right) \tan^{-1} \frac{d}{x} + \right. \\ \left. \frac{(c^2 + d^2)^{3/2} x}{12c^2 d^2} \tan^{-1} \frac{x}{(c^2 + d^2)^{1/2}} + \frac{c(d^2 + x^2)^{1/2}}{12} \left(\frac{1}{d^2} + \frac{1}{x^2} \right) \times \right. \\ \left. \tan^{-1} \frac{c}{(d^2 + x^2)^{1/2}} + \frac{d(c^2 + x^2)^{1/2}}{12} \left[\left(\frac{1}{c^2} + \frac{1}{x^2} \right) \tan^{-1} \frac{d}{(c^2 + x^2)^{1/2}} \right] \right] \quad (25)$$

The total value of K_6 can be written down but our interest actually is drawn to the form taken by equation (25) when d is small; calling $d^* = \sqrt{2} d$, the distance of closest approach, we get the simple result

$$K_6 \approx \frac{\pi c}{6d^*} \quad (26)$$

which shows the interesting result that the power dependence is down by a factor of one from equation (22) and agrees with the case of spheres at close separations. It is to be presumed that the configuration intermediate between equations (22) and (26) will depend on $d^{-3/2}$, where d is the closest distance of approach (cf. Vold, Fig. 1c).

* Notice that for $y = 0$ this equation becomes equivalent to equation (9).

¹ Kallmann, H., and M. Willstätter, *Naturwissenschaften*, **20**, 952 (1932).

² Hamaker, H. C., *Physica*, **4**, 1058 (1937).

³ de Boer, J. H., *Trans. Faraday Soc.*, **32**, 10 (1936).

⁴ Bradley, R. S., *Phil. Mag.*, **13**, 853 (1932).

⁵ Atoji, M., and W. N. Lipscomb, *J. Chem. Phys.*, **21**, 1480 (1953); Isihara, A., and R. Koyama, *J. Phys. Soc. Japan*, **12**, 32 (1957); DeRocco, A. G., *J. Phys. Chem.*, **62**, 890 (1958).

⁶ Dube, G. P., and H. K. Dasgupta, *Indian J. Phys.*, **13**, 411 (1939).

⁷ Bouwkamp, C. J., *Kon. Nederland Akad. Wetenschap.*, **50**, 1071 (1947).

⁸ The methods of this paper were also applied to the case of circles to give the result of Bouwkamp and of Dube and Dasgupta, which for two coplanar circles of radius R whose centers are separated by C is just $\pi^2 R^4 (C^2 - R^2) / C^3 (C^2 - 4R^2)^{3/2}$, apart from multiplication by a known energy constant.

⁹ Thomaes, G. J., *J. Chim. Phys.*, **49**, 323 (1952); Pitzer, K. S., *J. Am. Chem. Soc.*, **77**, 3427 (1955), incorrect beyond the second term; Lambert, J. A., *Australian J. Chem.*, **12**, 109 (1959).

¹⁰ Vold, M.

¹¹ Sparnaay

¹² Slater, J.

¹³ Neugebauer

¹⁴ Casimir,

¹⁵ DeRocco

¹⁶ Jehle, H.

MOUSE VIRUS

Much of the realm of specific virus on other aspects of basic problems in mouse hepatitis, reticulo-endothelial susceptibility, phages, infectious crosses, be inherited, the F₂ and

*Materia*ments of onto the compared acetate acid with amino-specified chick embryo Microbiology and 10 months text, chick. The chick in our laboratory in this manner.

The cultivation of the particles for titration

- ¹⁰ Vold, M. J., *J. Colloid Sci.*, **9**, 451 (1954).
¹¹ Sparnaay, M. J., *Recueil*, **78**, 680 (1959).
¹² Slater, J. C., and J. G. Kirkwood, *Phys. Rev.*, **37**, 682 (1931).
¹³ Neugebauer, Th., *Z. Physik*, **107**, 785 (1937).
¹⁴ Casimir, H. B. G., and D. Polder, *Phys. Rev.*, **73**, 360 (1948); *Nature*, **158**, 787 (1946).
¹⁵ DeRocco, A. G., (to be published).
¹⁶ Jehle, H., these PROCEEDINGS, **43**, 847 (1957).

MOUSE MACROPHAGES AS HOST CELLS FOR THE MOUSE HEPATITIS
 VIRUS AND THE GENETIC BASIS OF THEIR SUSCEPTIBILITY*

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Much of the mystery of pathogenesis of animal virus disease lies in the realm of specificity of virus effect on cell type and in the differing effect of a given virus on different hosts. Tissue culture, despite its great contributions to other aspects of cytopathology, has not so far furnished significant clues to these basic problems. We have recently found that an acute virus disease of mice, mouse hepatitis,^{1, 2} has a selective destructive effect for cells derived from the reticulo-endothelial system (macrophages) and that the apparent genetic difference in susceptibility of different strains of mice is reflected in the behavior of macrophages from these strains in tissue culture. Tests of hybrids resulting from crosses between resistant and susceptible strains indicate that susceptibility is inherited and that genetic segregation of susceptibility and resistance occurs in the F₂ and backcross generations.

Materials and Methods.—The macrophages were obtained by explanting fragments of liver from newborn (1–3 days old) mice into roller tubes either directly onto the glass or onto a reconstituted collagen substrate.^{3, 4} The collagen was prepared according to the method of Ehrmann and Gey by extraction of 0.1 per cent acetic acid, dialysis against distilled water, and reconstitution to an agar-like slant with ammonium hydroxide vapors. The supernatant medium, except in otherwise specified cases; consisted of 60 per cent Gey's balanced salt solution, 10 per cent chick embryo extract (50 per cent), and 30 per cent horse serum (obtained from Microbiological Associates), with 0.004 per cent phenol red, 100 units of penicillin, and 10 micrograms of streptomycin. In some experiments, as designated in the text, chicken serum or a combination of chicken serum and horse serum was used. The chicken serum was obtained from White Leghorns kept for routine bleeding in our laboratory. The cultures were incubated in a roller drum and maintained in this manner at 37°C throughout the experiments.

The cultures were inoculated with the virus three to four days following explantation. The medium was renewed every two to three days until the end of the particular experiment. Supernatant fluids were frozen at –40°C and reserved for titration in mice. The cells were observed directly in the roller tube and their