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CORRESPONDENCE

A uniformly moving edge dislocation in an elastic strip

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ABSTRACT

A Fourier-transform technique is presented for obtaining the displacement field of an edge dislocation that is moving in a finite-width strip having clamped boundaries. Known results are reproduced by this technique, and new solutions are obtained, which can be compared with those from atomistic models.

§ 1. INTRODUCTION

Because plastic flow occurs through the motion of dislocations, a detailed knowledge of dislocation structure and propagation is desirable (Nabarro 1967). Recent work in molecular dynamics, solving equations of motion for crystals containing a few thousand particles, makes it possible to treat individual atomic displacements for dislocations in motion (Moss, Hoover, Hoover and Ashurst 1977). In comparing these numerical calculations with the present predictions of continuum elasticity theory, it is necessary to use identical boundary conditions. Boundary conditions are significant in dislocation problems, because the displacement field diverges at large R in the absence of boundary constraints (Nabarro 1967, p. 57).

In this paper, we use a Fourier-transform technique to obtain the displacement field of an edge dislocation that is moving uniformly in a finite-width strip. The dislocation is centred between the strip boundaries, which are clamped. This technique can also be used to treat dislocations positioned asymmetrically between the strip boundaries and/or with other boundary conditions, e.g., traction-free.

§ 2. METHOD

In the absence of body forces, the fundamental equation of motion in linear elasticity theory is :

$$\eta\nabla^2\mathbf{R}+(\lambda+\eta)\nabla(\nabla\cdot\mathbf{R})-\rho\ddot{\mathbf{R}}=0,$$

where

$$\mathbf{R}=U\hat{\imath}+V\hat{\jmath}+W\hat{k}, \qquad (1)$$

and U, V and W are the x, y, z displacement-vector components, respectively, and λ and η are the Lamé constants. Equation (1) says that the net force on a volume element of material is zero. Love (1920) showed that a superposition of waves moving with a common velocity c would satisfy eqn. (1). We write

$$U=\int_0^\infty [A(k)\exp(-\gamma k|y|)\sin kx' + B(k)\phi\exp(-\phi k|y|)\sin kx']\,dk, \qquad (2)$$

$$V=\int_0^\infty [\operatorname{sgn}(y)A(k)\gamma\exp(-\gamma k|y|)\cos kx' + \operatorname{sgn}(y)B(k) \times \exp(-\phi k|y|)\cos kx']\,dk, \qquad (3)$$

$$W=0,$$

where $\phi=(1-c^2/c_T^2)^{1/2}$, $\gamma=(1-c^2/c_L^2)^{1/2}$, $x'=x-ct$, and $|c|<c_T$; $c_T=(\eta/\rho)^{1/2}$ and $c_L=(\lambda+2\eta/\rho)^{1/2}$ are the velocities of transverse and longitudinal waves in a medium of density ρ. If b = Burgers vector and

$$A(k)=\frac{-bc_T^2\operatorname{sgn}(y)}{\pi kc^2}, \quad B(k)=\frac{+bc_T^2\,\alpha^2\operatorname{sgn}(y)}{\pi c^2k\phi},$$

$\alpha^2=1-c^2/2c_T^2$, and one adds to U the function $b/4\operatorname{sgn}(y)$, the exact solution (Eshelby 1949) for a moving edge dislocation in the infinite x–y plane is obtained. In the limit of zero velocity, the solution reduces to the well-known static result,

$$U=\frac{bc_T^2}{\pi c^2}\left[-\tan^{-1}\frac{x'}{\gamma y}+\alpha^2\tan^{-1}\frac{x'}{\phi y}\right]+\frac{b}{4}\operatorname{sgn}(y)$$
$$\rightarrow\frac{b}{2\pi}\left[-\tan^{-1}\frac{x}{y}+\frac{xy}{2(1-\nu)(x^2+y^2)}\right]+\frac{b}{4}\operatorname{sgn}(y), \qquad (4)$$

$$V=\frac{bc_T^2}{2\pi c^2}\left[\gamma\ln(x'^2+\gamma^2y^2)-\frac{\alpha^2}{\phi}\ln(x'^2+\phi^2y^2)\right]$$
$$\rightarrow\frac{b}{2\pi}\left[\frac{2\nu-1}{4(1-\nu)}\ln(x^2+y^2)+\frac{y^2}{2(1-\nu)(x^2+y^2)}\right], \qquad (5)$$

$$W=0,$$

where $U(x<0,\pm0)=\pm b/2$, $U(x>0,0)=0$, and ν = Poisson's ratio.

The important physical feature of eqns. (2) and (3) is that an edge dislocation is composed of a unique admixture of wave-equation solutions. Furthermore, any convergent admixture of these solutions will satisfy eqn. (1). This property of the Fourier-transform solution allows one to consider various boundary value problems in a finite or semi-finite region of space. Consider the relaxation of an imposed shear displacement b by the passing of an edge dislocation centred in a finite-width strip. The boundary conditions are $U(x'<0,\pm0)=\pm b/2$, $U(x'>0,0)=0$, $U(x',\pm A)=\pm b/2$, $V(x',\pm A)=0$, where A = the strip half-width. We construct the displacement field by using an 'image' method (Moss and Hoover 1978) and write

$$\frac{\pi U c^2}{b c_T{}^2} = \int_0^\infty dk \frac{\sin kx'}{k} \Bigg[-\operatorname{sgn}(y') \exp(-\gamma k|y'|) + \operatorname{sgn}(y')\alpha^2 \exp(-\phi k|y'|)$$

$$+ \sum_{n=1}^{\infty} A_n \exp[\gamma k(y'-2n)] + \sum_{n=1}^{\infty} B_n \phi \exp[\phi k(y'-2n)]$$

$$+ \sum_{n=1}^{\infty} C_n \exp[-\gamma k(y'+2n)] + \sum_{n=1}^{\infty} D_n \phi \exp[-\phi k(y'+2n)] \Bigg]$$

$$+ \left[\frac{b}{4} \operatorname{sgn}(y') + \frac{by'}{4}\right] \frac{\pi c^2}{b c_T{}^2}, \qquad (6)$$

$$\frac{\pi V c^2}{b c_T{}^2} = \int_0^\infty dk \frac{\cos kx'}{k} \Bigg[-\gamma \exp(-\gamma k|y'|) + \frac{\alpha^2}{\phi} \exp(-\phi k|y'|)$$

$$+ \sum_{n=1}^{\infty} -A_n \gamma \exp[\gamma k(y'-2n)] + \sum_{n=1}^{\infty} -B_n \exp[\phi k(y'-2n)]$$

$$+ \sum_{n=1}^{\infty} C_n \gamma \exp[-\gamma k(y'+2n)] + \sum_{n=1}^{\infty} D_n \exp[-\phi k(y'+2n)] \Bigg], \qquad (7)$$

where $x' = (x-ct)/A$ and $y' = y/A$.

The coefficients are most easily represented as power series in $\exp[k(\gamma-\phi)]$ and $(1-\gamma\phi)^{-1}$:

$$A_n = \sum_{m=1}^{2n} \frac{a(m,n)}{(1-\gamma\phi)^n} \xi^{m-1}, \qquad (8)$$

$$B_n = \sum_{m=1}^{2n} \frac{b(m,n)}{(1-\gamma\phi)^n} \xi^{m-2n}, \qquad (9)$$

where $\xi = \exp[k(\gamma-\phi)]$.

The problem is solved by considering the boundary conditions and solving the resulting recursion formulae :

$$a(1,1) = 1+\gamma\phi, \quad a(2,1) = -2\alpha^2, \quad b(1,1) = -2\gamma, \quad b(2,1) = \frac{\alpha^2}{\phi}(1+\gamma\phi). \qquad (10)$$

$$b(m, n+1) = -2\gamma a(m,n) - b(m-2,n)(1+\gamma\phi). \qquad (11)$$

$$a(m, n+1) = [a(m,n) + b(m-2,n)\phi][1-\gamma\phi] - b(m,n+1)\phi. \qquad (12)$$

$$b(m-2,n) = 0, \quad \text{for} \quad m \leqslant 2. \qquad (13)$$

For this centred configuration with rigid boundaries $C_n = -A_n$, and $D_n = -B_n$.

Substituting eqns. (8) and (9) into (6) and (7), ~~respectively~~, and integrating

$$\frac{\pi U c^2}{b c_T^2} = -\tan^{-1}\frac{x'}{\gamma y'} + \alpha^2 \tan^{-1}\frac{x'}{\phi y'} + \sum_{n=1}^{\infty}\sum_{m=1}^{2n}\left[\frac{a(m,n)}{(1-\gamma\phi)^n}\right.$$

$$\times \tan^{-1}\left(\frac{2x'y'\gamma}{[(2n-y')\gamma+(\phi-\gamma)(m-1)]\times[(2n+y')\gamma+(\phi-\gamma)(m-1)]+x'^2}\right) + \frac{b(m,n)\phi}{(1-\gamma\phi)^n}$$

$$\left.\times \tan^{-1}\left(\frac{2x'y'\phi}{[(2n-y')\phi+(\phi-\gamma)(m-2n)]\times[(2n+y')\phi+(\phi-\gamma)(m-2n)]+x'^2}\right)\right]$$

$$+\left[\frac{b}{4}\operatorname{sgn}(y') + \frac{by'}{4}\right]\frac{\pi c^2}{b c_T^2};$$

$$\frac{\pi V c^2}{b c_T^2} = \frac{\gamma}{2}\ln(x'^2+\gamma^2 y'^2) - \frac{\alpha^2}{2\phi}\ln(x'^2+\phi^2 y'^2)$$

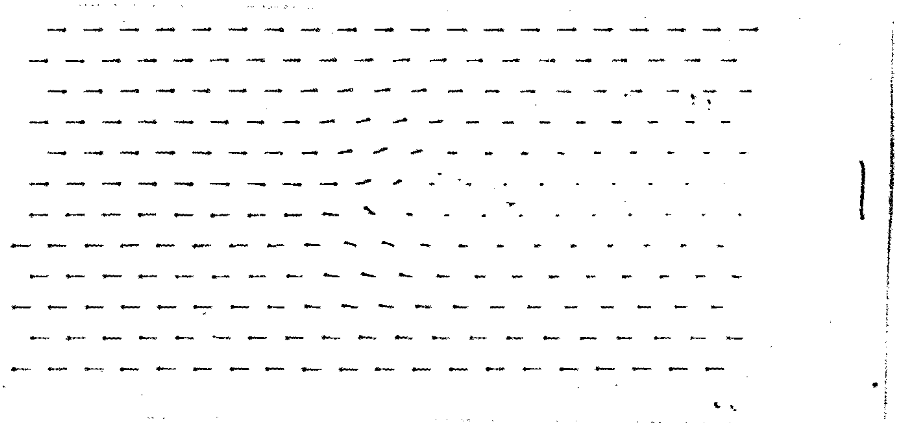
$$+\sum_{n=1}^{\infty}\sum_{m=1}^{2n}\left[\frac{\gamma a(m,n)}{2(1-\gamma\phi)^n}\ln(x'^2+[(2n-y')\gamma+(\phi-\gamma)(m-1)]^2)\right.$$

$$+\frac{\gamma a(m,n)}{2(1-\gamma\phi)^n}\ln(x'^2+[(2n+y')\gamma+(\phi-\gamma)(m-1)]^2)$$

$$+\frac{b(m,n)}{2(1-\gamma\phi)^n}\ln(x'^2+[(2n-y')\phi+(\phi-\gamma)(m-2n)]^2)$$

$$\left.+\frac{b(m,n)}{2(1-\gamma\phi)^n}\ln(x'^2+[(2n+y')\phi+(\phi-\gamma)(m-2n)]^2)\right].$$

Fig. 1

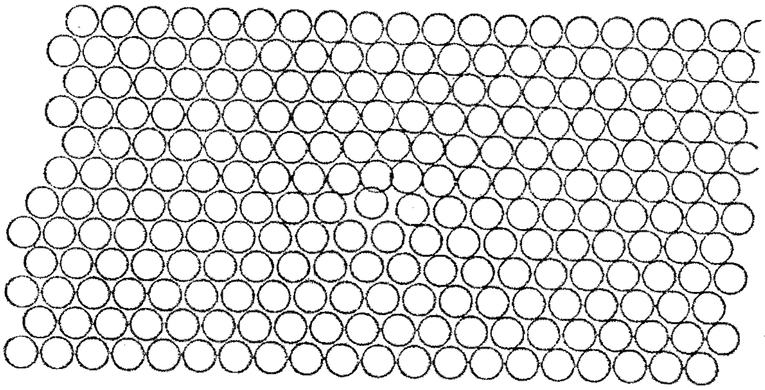


Displacements in a rigid-boundary strip after summing 40 images. The velocity is 0·56 c_T. The displacements are shown at the sites of a triangular lattice in order to visualize the effect of a finite lattice on the field. The tails of the arrows represent the atoms in an unstrained lattice; the heads represent present atomic sites.

§ 3. Results

All the calculations have been made with Poisson's ration $=0{\cdot}25$, $c_T=(3/8)^{1/2}$, $b=1$, and $C_L=(9/8)^{1/2}$. These same values are appropriate to the molecular-dynamics calculations (Hoover, Ashurst and Olness 1974) and do not affect the generality of the solution. Figure 1 shows the displacement field throughout the strip after taking 40 images. The solution has been

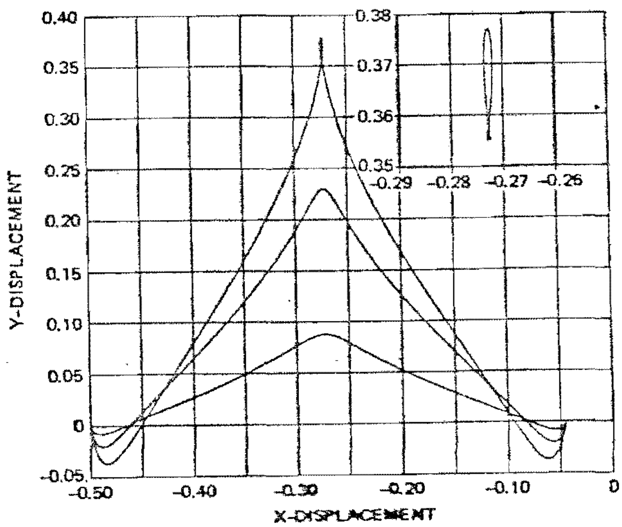
Fig. 2



2

Atomic view of continuum displacements in a rigid-boundary strip. Circles have been drawn around the heads of the arrows in fig. 1.

Fig. 3



3

Motion of the point $(x, y)=(0, -1/11)$ as the dislocation passes above it. The motion of the point is from right to left, and the y-displacement is plotted against the x-displacement. The motion increases with velocity, $c=0$, $0{\cdot}56$, and $0{\cdot}82\,c_T$. The plot in the upper right is an expanded view of the top of the $c=0{\cdot}82\,c_T$ curve.

checked on the boundaries and is accurate to within 1% for $|x'|<2$. The velocity is 0·56 c_T and $t=0$ (the dislocation is located at $(x, y)=(0, 0)$). An atomic representation of the dislocation is shown in fig. 2, where circles have been drawn around the heads of the arrows in fig. 1. Figure 3 shows the motion of the point $(x/A, y/A)=(0, -1/11)$ as the dislocation passes above it. The motion of the point is from right to left. Alternatively and equivalently, one could plot the displacements of the points $(x', -1/11)$ at a fixed time. Velocities of 0, 0·56, and 0·82 c_T are shown. The motion increases with velocity. An expanded view of the top of the $c=0{\cdot}82\ c_T$ curve is shown in the upper right of fig. 3. This same behaviour appears in Eshelby's infinite plane solution. A detailed comparison of this solution with the results of the molecular dynamics calculations is in progress (W. C. Moss 1978, unpublished).

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