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Heat flux at the transition from harmonic to chaotic flow in thermal convection

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Numerical simulations of the fully compressible Navier-Stokes equations are used to study the transition from simple-periodic "harmonic" thermal convection to chaotic thermal convection as the Rayleigh number Ra is increased. The simulations suggest that a sharp discontinuity in the relationship between the Nusselt number Nu (the ratio of the total heat flux to the Fourier heat flux) and the Rayleigh number is associated with this transition in flow morphology. This drop in the Nusselt number is also seen in the data reported in independent experiments involving the convection of two characteristically different fluids—liquid mercury [Phys. Rev. E **56**, R1302 (1997)] (a nearly incompressible fluid with Prandtl number Pr=0.024) and gaseous helium [Phys. Rev. A **36**, 5870 (1987)] (a compressible fluid with unit Pr). The harmonic flow generates a dual-maximum (quasiharmonic) temperature histogram, while the chaotic flow generates a single-maximum histogram at the center point in the simulated cell. This is consistent with the temperature distributions reported for the convecting mercury before and after the drop in Nu. Our simulations also suggest a hysteresis in the Nu-Ra curve linking the two distinctly different flow morphologies, heat fluxes, and temperature-fluctuation histograms at the same Rayleigh number. [S1063-651X(98)01909-6]

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I. INTRODUCTION

In the well-known "Rayleigh-Bénard" problem, a viscous, heat-conducting fluid enclosed by thermal boundaries in a gravitational field makes a transition from quiescent Fourier heat conduction to steady convection at a critical Rayleigh number ($Ra_c = 1708$ with the Boussinesq approximation [1]). The steadily convecting flow transports heat more effectively. At a much higher Rayleigh number, the system makes a second transition, from steady convection to time-dependent convection. The previously steady convection rolls start to oscillate vertically. Observables, such as the heat flux and the position of the convecting rolls, vary periodically (harmonically) in time [2,3]. Eventually, this periodic motion gives way to chaotic flow as vertical plumes start to influence the flow. Observables lose their simple time dependence, and the flow becomes irregular.

Experiments with thermally convecting mercury, a low-Prandtl-number fluid, reported in Ref. [4], show that a well-known power law for "hard" turbulent convection, relating the Rayleigh number to the dimensionless heat flux, $Nu \sim Ra^{2/7}$, persists after the inversion of the thermal and viscous boundary layers. Theories predicting this $\frac{2}{7}$ power law [5,6] are based on the assumption that the thermal boundary layer is purely diffusive and confined within the viscous boundary layer. The results of these experiments contradict the basic assumption of the theories.

An interesting feature of the reported data is a "bump" in the Nusselt-number–Rayleigh-number relation showing that the Nusselt number drops with increasing Rayleigh number. This drop in Nusselt number is accompanied by an apparent change in flow morphology, indicated by the temperature-fluctuation histogram at a probe fixed at the center of the cell. The temperature histogram goes from a dual-maximum profile before the Nusselt number drop to a single-maximum profile, and has been interpreted, in the absence of the ability to visualize the actual flow, as a change in the number of convection rolls in the system.

Another set of experiments, involving the convection of gaseous helium [7], describes transitions in the flow morphology for Rayleigh numbers from 10^3 to 10^{11}. The onset of the "oscillatory" convecting flow was reported for $Ra = 9\times10^4$. The onset of the "chaotic" flow was reported for $Ra = 1.5\times10^5$ and continues to $Ra = 2.5\times10^5$. A drop in the Nusselt number is seen in the data for the transition from the oscillatory to chaotic flows. At much higher Rayleigh numbers, the "hard-turbulence" state is reached, and the scaling relation $Nu-1\sim Ra^{0.282}$ is reported.

Both experiments were conducted in cylindrical vessels. The vessel for the helium gas had an aspect ratio of 1 (equal height and width). The mercury experiments were conducted in various aspect ratio vessels, but the set of data that spans the transition from harmonic to chaotic flow was from a vessel with an aspect ratio of 2 (twice as wide as high).

II. RESULTS

Our simulations also reveal that a drop in the Nusselt number, as the Rayleigh number is increased, is associated with the transition in the flow morphology from "periodic" to "chaotic" convection (see Fig. 1). Computer simulations offer the ability to visualize the time dependence of any variable field, such as the temperature or velocity, in order to characterize the flow. Time-averaged quantities, such as the heat flux, computed from simulations of increasing time and space resolution, are used to extrapolate a value for the continuum (zero-mesh) limit. Steady-state convection is observed for Rayleigh numbers up to 9×10^4. For slightly greater Ra, the simple-periodic "harmonic" flow is observed. This is consistent with the results of experiments with helium [7] and *ab initio* molecular dynamics simulations [2]. The heat flux for this flow varies in time with a

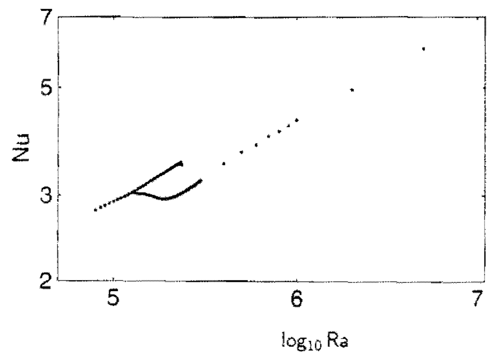

FIG. 1. The Nusselt number for convecting flows of various Rayleigh numbers. Within the hysteretic range, two flow morphologies are possible, each corresponding to a different Rayleigh number. For example, at $\mathrm{Ra}=2\times 10^5$, a harmonic flow with Nu $=3.392$ and a chaotic flow with $\mathrm{Nu}=2.959$ are both stable possibilities.

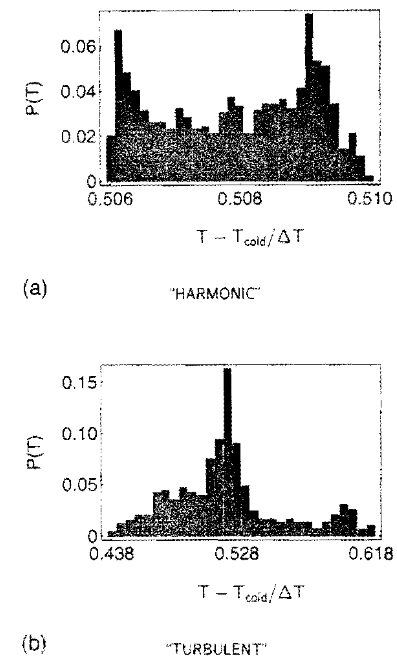

FIG. 2. The temperature-fluctuation histogram for a point at the center of the simulated cell for the (a) "harmonic" and (b) "chaotic" flows.

single frequency, equal to the frequency of the vertical oscillation of the rolls. This is the second characteristic frequency of the system—the first being the frequency that a volume of fluid travels around a convecting roll. This harmonic flow is stable for $9\times 10^4>\mathrm{Ra}>2.4\times 10^5$. For slightly higher Rayleigh numbers, at least one additional characteristic frequency is introduced, that of cold, downward-flowing and warm, upward-flowing plumes sweeping horizontally back and forth. The time dependence of the heat flux for this flow is more complicated, and the time average is roughly 10% l[illegible] This three-period flow is less efficient at transporting he[illegible] since the plumes disturb the opposite thermal boundary layer and sweep material in a direction that is counter to the flow of heat. The maximum Lyapunov exponent (for our discrete approximation) is greater for this flow, as expected for a "chaotic" system. This route to chaos, observed in our simulations, is consistent with Ruelle's idea that if three incommensurate frequencies simultaneously exist in a system, regular motion becomes highly unstable in favor of motion on a strange attractor (chaotic motion) [8]. We also find that, for the two-period flow, the temperature histogram at a point in the center of the simulated cell is harmoniclike, having two maximum corresponding to the "turning-point" temperatures. For the three-period (chaotic) flow, the temperature histogram at the same point has a single maximum in the center of the temperature range (see Fig. 2). This is consistent with the temperature histogram for turbulent flows [1,2,9].

The computer simulations show that chaotic flow is possible for $\mathrm{Ra}>1.3\times 10^5$. This suggests that this drop in the Ra-Nu relation is an hysteretic link between two-period (harmonic) and three-period (chaotic) flows. It is possible that two systems, with the same Rayleigh number, have different flow morphologies, heat fluxes (and Nu), and temperature-fluctuation histograms. The coexistence of different flow morphologies at the same Rayleigh number has been seen in si[illegible]tions of compressible fluids [10], and using the Boussin[illegible]pproximation of a convecting system [11].

III. METHODS

The method used to study the transition to turbulent convection involves numerically solvi[illegible] the fully compressible Navier-Stokes equations for a two-dimensional ideal gas, $P_{\mathrm{eq}}=\rho k_B T=\rho e$, enclosed between two rigid thermal boundaries separated by a distance L, and in the presence of a body force g. The sides boundaries are periodic and have a length scale corresponding to a cell with an aspect ratio of 2. By considering units such that the Boltzmann constant, the mean density, and the heat capacity are set to unity, the Rayleigh number for the system is defined as $\mathrm{Ra}=\alpha g\Delta T L^3/\eta\kappa$, where α is the thermal expansion coefficient, and η and κ are shear viscosity and heat transfer coefficients. Since, for an ideal gas, $\alpha=T^{-1}$, and the body force can be assigned a magnitude such that a small volume element of fluid moving from the lower high-temperature boundary to the upper low-temperature boundary gains a potential energy to exactly compensate for the loss in thermal energy ($g=k_B\Delta T/\rho_0 L=\Delta T/L$, setting $k_B=\rho_0=1$), the Rayleigh number can be written as $\mathrm{Ra}=\Delta T^2 L^2/T\eta\kappa$. The Prandtl number Pr (the ratio of the kinematic viscosity to the thermal diffusion coefficient) is set to unity. The maximum velocity of the flows were always less than half of the sound speed, so shock waves do not influence the results.

For each data point, the flow is allowed to develop from the initial conditions for several thousand sound-traversal times. At this point, calculations of the heat flux and temperature histograms are carried out for several thousand more sound-traversal times. To demonstrate the existence of the hysteresis, the initial state of a run is set to a state of the well-developed flow of a run with a Rayleigh number that is different by 5000 (about 3% of the range). The Rayleigh number is varied in this study by changing either the transport coefficients (which also varies the diffusion-traversal times), the mean temperature (which also varies the sound-traversal time and the thermal expansion), or the length scale (which varies the diffusion- and sound-traversal times). In each case, the results are qualitatively the same. The stability of the well-developed flows is tested by introducing a random noise with a magnitude equal to roughly 10% of the mean value of each state variable, and allowing the system to continue to develop. The stability test demonstrates that two stable, but different, flows are possible for the same Rayleigh number in this range.

IV. CONCLUSIONS

Although our simulations are not an explicit attempt to model the experiments with convecting mercury or helium, the character of the results is very similar. Since the simulations use an ideal gas equation of state, model a compressible fluid, and have a unit Prandlt number, it is a fair model for the helium gas experiment. On the other hand, the simulations are two dimensional (2D). The flows in a 2D system are different from those of a 3D system. The simulations are not very good models for the mercury experiments—2D rather than 3D, compressible rather incompressible, and intermediate rather than low Pr. Despite the difference, the behaviors in all three situations have a common characteristic—a drop in the heat flux as the system makes the transition to chaotic flow.

Since the Nusselt number is identically equal to the dimensionless entropy production of the system, the results suggest that this dynamical system, driven farther from equilibrium, has a sudden drop in the entropy production as it makes a transition to chaos. The maximum Lyapunov exponent for the simulation is a measure of the rate at which phase-space information is lost. The drop in entropy production for the system at the transition is accompanied by a corresponding increase in this rate of information loss.

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